Onset of Benard-Marangoni convection in a rotating liquid layer with nonuniform volumetric energy sources

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The criteria for the onset of natural convection in a rotating liquid layer with nonuniform volumetric energy sources from absorbed thermal radiation are determined via linear stability analysis. The linearized perturbation equations are solved by using a numerical technique to obtain the eigenvalues that governs the onset of convection in a microgravity environment. The stability criteria are obtained in terms of the Marangoni number as function of the optical thickness. The influences of the Rayleigh number, Taylor number, Bond number, Crispation number, and Biot number on convection are examined in detail. These parameters provide a relationship between the critical Marangoni number and the Coriolis force, the buoyancy force, the interfacial tension, and the heat transport mechanisms.

Keywords: Benard-Marangoni convection; onset of convection; linear stability analysis

Introduction

Convective motion in a horizontal liquid layer driven by surface tension gradients and buoyancy forces has attracted great attention in the last several decades. Thermal convection in such a configuration can be found in many physical phenomena and various engineering applications in modern technology. These include many chemical processes (Davenport and King 1973), crystal growth (Chang and Wilcox 1975; Cutler 1977; Schwabe 1981), materials processing in space (Carruthers 1977; Chun 1980), geological (Knopoff 1969) and astrophysical (Tritton 1975) systems. The study of the mechanism of controlling undesirable convective motion generated in a fluid layer by both the buoyancy and surface tension forces in a microgravity environment has received a great deal of interest due to its application to the possibility of producing various new materials (Regel' 1988; Saghir 1988). Some notable investigations of this problem have been undertaken by Pearson (1958), Nield (1964), Scriven and Sternling (1964), Smith (1966), Debler and Wolf (1970), Davis and Homsy (1980), and Lebon and Cloot (1981). Numerous factors affect the initiation of convective instability, such as shear stress, stratification, phase change, chemical reaction, electric and magnetic fields, rotation, gravity, and interfacial tension. The effect of the stabilization action of the Coriolis force due to rotation has been examined by Niiler and Bisshopp (1965), Vidal and Acrivos (1966), Namikawa et al. (1970), Friedrich and Rudraiah (1984), and Sarma (1979, 1985a, 1985b, 1987).

With the exception of the limited results presented by Friedrich and Rudraiah (1984), the temperature gradient across the fluid layer is assumed to be a constant in many of these aforementioned studies. They have examined this problem without the gravity effect for four distinct temperature profiles corresponding, respectively, to a liquid layer heated from below, a liquid layer cooled from above, and liquid layers with constant internal heat generation (which include a parabolic and an inverted parabolic profiles). However, the study assumed a zero interfacial curvature in the local balance conditions for the thermal flux, and tangential and normal stresses. In most physical problems, a flat two-fluid interface and a constant temperature gradient across the liquid layer generally do not occur. In the present study, these restrictions are relaxed.

This paper is aimed at studying the onset motion of Benard–Marangoni convection in a microgravity environment ($10^{-6} - 10^{-3}g$). In contrast to earlier treatments, it is assumed that thermal convection is induced by both the buoyancy forces within the fluid and by temperature variations of the surface tension at the two-fluid interface; moreover, the basic unperturbed temperature gradient across the liquid layer is no longer uniform. The nonuniformity is a result of absorption and penetration of external radiation in the liquid medium. The basic unperturbed temperature gradient in the liquid layer is essentially exponential and increases monotonically from the lower to the upper boundaries. The critical Marangoni number and wave number are determined in order to examine the influences of the nonuniform volumetric energy sources, the rotation constraint, the buoyancy and interfacial tension mechanisms on Bernard–Marangoni convection.

The present paper investigates the gravity and surface tension-driven convection in a rotating single-fluid layer with a free deformable surface with respect to time-independent perturbations. The study considers the case of convection with a nonzero disturbed wavy interfacial curvature at the upper free two-fluid boundary. Oscillatory instability (Sarma 1985a) may occur when deformations are allowed at the interface. However,

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the work also provides evidence to support the view that the incipient instability occurs in the stationary form rather than in the oscillatory modes for liquid layer with small rotation. The present study, therefore, focuses the attention only on stationary instability. Furthermore, similar to most of the previous works dealing with the effect of surface deformation in thermogravity and thermocapillary convection, the principle of exchange of stabilities is assumed valid without proof (Zeren and Reynolds 1972; Cloot and Lebon 1985; Sarma 1985b, 1987; Garcia-Ybarra et al. 1987).

In the following section, the disturbance equations for the study of onset of natural convection in a rotating liquid layer with internal nonuniform volumetric energy sources are presented. The nonlinear temperature across the fluid layer is of particular interest in this study; therefore, a comprehensive discussion of the steady-state temperature profile is discussed in detail. The equations governing the criteria for the onset of convection are solved numerically, and the eigenvalues for the stability problem are obtained by using the Chebyshev pseudospectral method. The solutions give the threshold values for the onset of stationary instability in terms of the Marangoni number and the corresponding wave number. In the final section, the combined effects of all the relevant influential parameters of the problem (e.g., optical thickness of the liquid layer, the Biot, Bond, Crispation, Rayleigh, and Taylor numbers) on these two critical numbers are examined.

Formulation of the problem

The geometry for the physical problem is shown in Figure 1. We consider an incompressible liquid layer of infinite horizontal extent which is confined in the region $0 \leq z \leq d + \eta^*$, where d is the mean thickness and $\eta^*(x, y, t)$ is the local deflection of the upper surface from the mean. The lower rigid boundary is maintained at a constant temperature, while the upper boundary is exposed and exchanges heat with the ambient. The z-axis is taken as the vertical coordinate (perpendicular to the free surface) with the origin at the bottom rigid surface, and the x-axis is the horizontal coordinate (along the surface). The liquid is heated

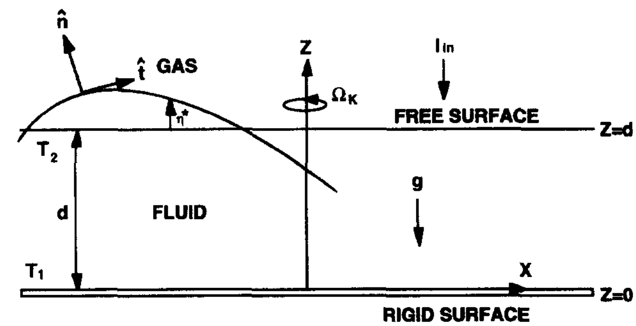


Figure 1 Schematic diagram of the physical system

Notation

Bi	Biot number, hd/k
Bo	Bond number, $\rho g d^2/\sigma_o$
Cr	Crispation number, $\mu_o \alpha/\sigma_o d$
C_p	specific heat at constant pressure, J/kg-K
d	mean thickness of the fluid layer, m
D	differential operator, d/dZ
$f(Z)$	dimensionless steady-state temperature gradient
g	gravitational acceleration in the z-direction, m/s^2
h	convective heat transfer coefficient at the fluid–air interface, W/m^2-°C
I_{in}	radiation intensity incident at the fluid–air interface, W/m^2-sr
I_{tr}	radiation intensity transmitted into the fluid, W/m^2-sr
k	thermal conductivity, W/m-K
Ma	Marangoni number, $(-\partial\sigma/\partial T)_o(\Delta T d/\mu_o \alpha)$
$\mathbf{n}$	normal unit vector at free upper surface
Q^r	steady-state volumetric heat generation, W/m^3
Ra_e	external Rayleigh number, $g\beta\Delta T d^3/\nu\alpha$
Ra_i	internal Rayleigh number, $g\beta I_{tr} d^4/\nu\alpha k$
R^*	ratio of external to internal Rayleigh number, $\mathrm{Ra}_e/\mathrm{Ra}_i$
$\mathbf{t}$	tangential unit vector at free upper surface
T	temperature, K
T^*	temperature disturbance, K
ΔT	temperature difference $(T_1 - T_2)$, K
Ta	Taylor number $2\Omega_k d^2/\nu_o$
T_∞	ambient temperature, K
w^*	velocity disturbance in the z-direction, m/s
W	dimensionless amplitude function of the velocity disturbance in the z-direction, $W = w^* d/\alpha$
z	Cartesian coordinate in the z-direction, m
Z	dimensionless depth in the z direction, z/d

Greek

α	thermal diffusivity, m^2/s
β	thermal expansion coefficient, K^{-1}
η	dimensionless interfacial deflection, η^*/d
η^*	local deflection of the free surface from the mean, m
Θ	dimensionless amplitude function of the temperature disturbance, $\Theta = T^*/\Delta T$
κ	extinction coefficient, m^{-1}
μ	dynamic viscosity, kg/m-s
ν	kinematic viscosity (μ/ρ), m^2/s
ρ	density, kg/m^3
σ	surface tension, kg/s^2
τ	optical thickness, κd
τ_{eff}	effective transmittance of the upper surface
ϕ^*	vorticity disturbance in the z-direction, rad/s
Φ	dimensionless amplitude function of the vorticity disturbance, $\Phi = \phi^* d^2/\alpha$
Ω_k	constant angular velocity about the z-axis, rad/s

Subscripts

b	undisturbed state
o	reference state
1	lower boundary
2	upper boundary
c	critical
e	external
i	internal
$\mathbf{k}$	unit vector in z direction

by a nonuniform volumetric energy source of strength Q^r, due to external radiation incident at the upper free surface. The liquid layer is rotating about the vertical z-axis with a constant angular velocity Ω_k. In the formulation, the Boussinesq approximation is used by assuming that all fluid properties, such as viscosity, thermal conductivity, specific heat, thermal expansion coefficient, are independent of temperature with the exception of the density in the body force term in the equation of momentum. Additionally, the surface tension is allowed to vary linearly with temperature.

It is interesting to investigate whether the nonlinear temperature profile, as a result of the external incident thermal radiation, can be maintained across the fluid layer without leading to convective motion. Therefore, the main purpose of this study is to formulate a linear stability problem using the conventional normal mode procedure and determine the condition for the onset of convection. Based on the conventional linear stability theory outlined by Chandrasekhar (1961), it is assumed that the field variables undergo infinitesimal disturbances. The linearized dimensionless equations governing the z-component of the velocity, the vorticity, and the temperature perturbations have been developed by many investigators in the past. Here we provide only the applicable governing disturbance equations for the current study without repetition. The interested readers should refer to the studies by Sarma (1985a, 1985b, 1987) for more details. Note that the disturbance equations given in the aforementioned references are derived for only a linear temperature profile, $f(Z)=1$; therefore, these equations have been modified to accommodate the nonlinear temperature profile as required in this study.

Disturbance equations

The equations governing the marginal state are given as

$$(D^2-a^2)^2W-\mathrm{Ta}\cdot DW=\mathrm{Ra}_i\cdot a^2\Theta \tag{1}$$

$$(D^2-a^2)\Theta=-f(Z)\cdot W \tag{2}$$

$$(D^2-a^2)\Phi=-\mathrm{Ta}\cdot W \tag{3}$$

Subject to the following boundary conditions:

at $Z=0$:

$$W=0 \tag{4}$$

$$DW=0 \tag{5}$$

$$\Theta=0 \tag{6}$$

$$\Phi=0 \tag{7}$$

at $Z=1$:

$$W=0 \tag{8}$$

$$D\Phi=0 \tag{9}$$

$$D\Theta+\mathrm{Bi}\cdot[\Theta-f(1)\cdot\eta]=0 \tag{10}$$

$$(D^2-a^2)W+\mathrm{Ma}\cdot a^2\cdot[\Theta-f(1)\cdot\eta]=0, \tag{11}$$

$$\mathrm{Cr}\cdot(D^3W-3\cdot a^2\cdot DW)-a^2\cdot(a^2+\mathrm{Bo}+\mathrm{Ra}_i\cdot\mathrm{Cr})\cdot\eta=0 \tag{12}$$

Equations 4–9 are the familiar boundary conditions of the classical Benard problem (Chandrasekhar 1961). They represent the velocity, thermal, and vorticity conditions at the boundaries. Equation 10 is the interfacial thermal condition which denotes the fluid medium that exchanges heat with the environment. Boundary conditions 11 and 12 express the continuity of tangential and normal stress at the interface.

The parameters that control the stability of the fluid layer include the wave number a, Biot number Bi, Bond number Bo, Crispation number Cr, Marangoni number Ma, Taylor number Ta, optical thickness τ, external and internal Rayleigh numbers Ra_e and Ra_i. The relationship between the initial temperature profile and the Rayleigh numbers are shown later in this paper. The Biot number denotes the effect of heat transfer by conduction to convection. The Bond number represents the measurement of the effect of interfacial gravity waves. The Crispation number is related to the degree of deformability of the upper surface. The Marangoni number measures the variation of the surface tension with the temperature. The Taylor number represents the ratio of the Coriolis force to the viscous frictional force. The Rayleigh number is a measurement of the buoyancy force and the viscous force.

The disturbance Equations 1–3, together with the homogeneous boundary conditions 4–12 constitute an eigenvalue problem. The nontrivial functions W, Θ, and Φ satisfy all of these conditions only if there exists a functional relationship so that

$$F(a,\mathrm{Bi},\mathrm{Bo},\mathrm{Cr},\mathrm{Ma},\mathrm{Ra}_e,\mathrm{Ra}_i,\mathrm{Ta},\tau)=0 \tag{13}$$

The primary objective of this investigation is to determine the critical value of the Marangoni number and the corresponding wave number on the locus of states neutrality stable represented by the parametric space given by Equation 13. Also, the nonlinear temperature profile $f(Z)$ across the liquid layer needs to be examined and its effects on the convective motion need to be exploited.

Steady-state temperature profile analysis

In Equation 2, $f(Z)$ is the dimensionless temperature gradient across the liquid layer at the equilibrium condition. We assume that the temperature is uniform and constant throughout the boundary. The temperatures of the lower and upper boundary surfaces are designated as T_1 and T_2, respectively. Thermal radiation with intensity I_{in} is normally incident at the interface. The liquid layer is assumed to be an absorbing and nonscattering medium. Emission is assumed to be negligible. The portion of the incident thermal radiative intensity that has penetrated and transmitted in the fluid layer is

$$I_{\mathrm{tr}}=I_{\mathrm{in}}\cdot\tau_{\mathrm{eff}} \tag{14}$$

where τ_{eff} is the effective transmissivity of the interface surface. Thus, the amount of incident thermal radiation that has penetrated to a depth $(d-z)$ can be written as (Yucel and Bayazitoglu 1979; Lam and Bayazitoglu 1988)

$$Q^r(z)=I_{\mathrm{tr}}\cdot\kappa\cdot\{\exp[-\kappa\cdot(d-z)]\} \tag{15}$$

The volumetric rate of heat generation Q^r, therefore, refers to that of the incident thermal radiation being penetrated and absorbed in the fluid layer. By incorporating the rate of volumetric heat generation in the governing steady-state energy equation, one can solve for the steady-state temperature of the conduction regime from

$$\frac{d^2T_b}{dz^2}+\frac{I_{\mathrm{tr}}\cdot\kappa\cdot\{\exp[-\kappa\cdot(d-z)]\}}{k}=0 \tag{16}$$

subject to the boundary conditions

$$T_b(0) = T_1 \tag{17}$$

$$T_b(d) = T_2 \tag{18}$$

The solution of the above system yields the steady-state temperature profile

$$T_b = T_1 - (T_1 - T_2)\left(\frac{z}{d}\right) - \left(\frac{I_{tr}}{k\cdot\kappa}\right)\left\{\exp[-\kappa\cdot(d-z)] + [1-\exp(-\kappa\cdot d)]\left(\frac{z}{d}\right) + \exp(-\kappa\cdot d)\right\} \tag{19}$$

and the dimensionless steady-state temperature gradient across the fluid layer takes the form

$$f(Z) = -\frac{d}{\Delta T}\frac{dT_b}{dZ} = 1 + R^*\cdot\left\{\exp[-\tau\cdot(1-Z)] - \frac{[1-\exp(-\tau)]}{\tau}\right\} \tag{20}$$

where

$$R^* = \frac{\mathrm{Ra}_e}{\mathrm{Ra}_i}$$

The dimensionless steady-state conduction temperature gradient (Equation 20) is shown in Figures 2a and b for $\tau = 1$ and 10 and various values of the dimensionless R^*. If there is no external thermal radiation incident at the upper boundary, $R^* = 0$, Equation 20 reduces to

$$f(Z) = 1 \tag{21}$$

which represents a linear temperature profile across the fluid layer due to heating from below.

Solution of the eigenvalue problem

The interfacial deflection η can be eliminated from the differential systems by the use of Equation (12). The stability problem is solved for a given steady-state conduction temperature gradient across the liquid layer by specifying the values of Bi, Bo, Cr, Ra_e, Ra_i, and Ta for various values of the optical thickness τ. The aim of the problem is to minimize the Marangoni number and the corresponding wave number for these particular physical parameters.

The mathematical model in terms of the disturbance equations for the stability problem is represented by an eighth-order system of differential Equations 1–3. The eigensystem can be solved by the finite-difference or the Fourier–Galerkin methods. Due to the complexity of the basic flow (Equation 20), a numerical method proposed by Yang (1990) is selected for determining the criteria for the onset of convection in this study. The technique is based on the Chebyshev pseudospectral method (Canuto et al. 1988), which employed the Chebyshev polynomials and a special transformation function to convert the original differential eigensystem to an algebraic eigensystem. The resulting generalized eigensystem is then solved directly by using the *QZ* algorithm (Moler and Stewart 1973), which is available in the IMSL package (Rice 1983) under the special routine named EIGZC.

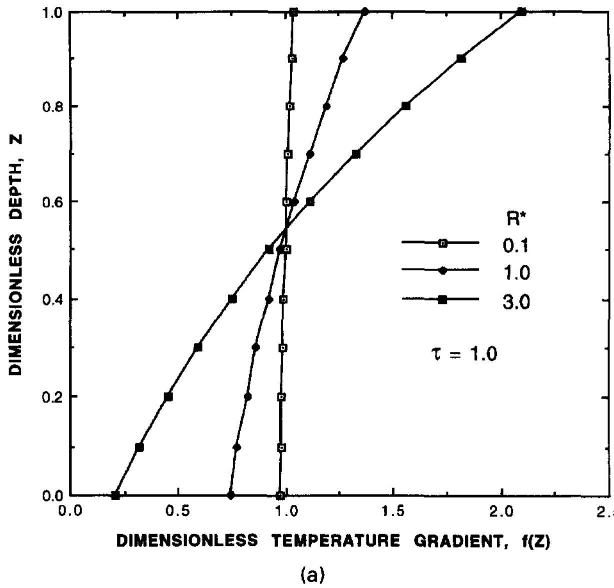


Figure 2a Dimensionless steady-state conduction temperature gradient

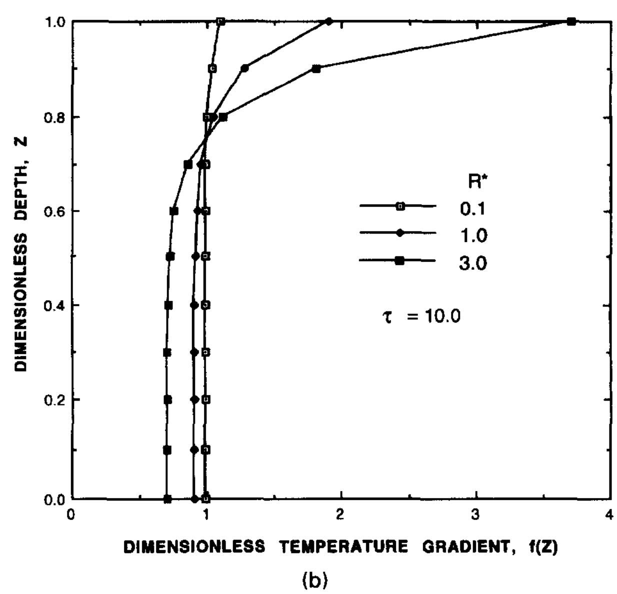


Figure 2b Dimensionless steady-state conduction temperature gradient

Results and discussion

The criteria for the onset of Benard–Marangoni convection in a horizontal rotating fluid layer subjected to external thermal irradiation at the upper interface have been obtained by using the Chebyshev pseudospectral method. Recent studies in this similar topic (Sarma 1985a, 1985b, 1987) showed that convective instability occurs in stationary rather than in oscillatory modes under certain conditions. The current study focuses only on the stationary mode of convective instability with finite waves. In view of the large number of characteristic dimensionless parameters, a representative order of magnitude for these parameters similar to those used by Sarma (1985b, 1987) and Sreenivasan and Lin (1978) were used in this study. As stated in the study performed by Sarma (1985b), the values of these parameters can be regarded as representative for experiments in an Earth laboratory on thin liquid layers ($d \sim$ mm) and experiments in an orbital laboratory with thicker layers ($d \sim$ cm, $g \sim 10^{-4} g_{\text{earth}}$) for metallic and semiconductor melts and silicone oils. The variations in the critical Marangoni number Ma_c with the optical thickness of the fluid layer are presented in this paper. The effects of internal heat generation, Coriolis force, buoyancy, and interfacial tension mechanisms on the critical Marangoni are subsequently discussed.

To establish the accuracy of the Chebyshev pseudospectral method, a comparison with the results of previous studies was made. For a similar geometric configuration with the conditions of $\text{Cr} = \text{Bi} = \text{Bo} = R^* = \text{Ta} = 0$, the present method predicted 79.607 and 1.993 for Ma_c a_c, respectively. Nield (1964) obtained identical results by using the Fourier series expansion technique. The agreement of these results substantiates the applicability of the Chebyshev pseudospectral method for determining the conditions leading to the onset of convective motions in a liquid layer.

The goal of the present study is to investigate the effect of nonuniform temperature gradients on Benard–Marangoni instability of a horizontal liquid layer. The threshold condition leading to the onset of convective motion is designated by the critical Marangoni number. The variation of Ma_c with the optical thickness τ for various values of the Rayleigh, Taylor, Bond, Crispation, and Biot numbers is presented. Note that the optical thickness τ is defined as the product of the absorption coefficient and the thickness of the liquid layer for a nonscattering medium.

As mentioned previously, the nonuniformity in the steady-state conduction temperature gradient across the liquid layer is due to the absorption of external thermal irradiation. The effects of thermal irradiation can be explained by referring to the steady-state temperature gradient across the liquid layer, as shown in Figures 2a and 2b. The steady-state dimensionless temperature gradients $f(Z)$ are presented for various values of the external and internal Rayleigh number ratios ($R^* = \text{Ra}_e/\text{Ra}_i$) for $\tau = 1$, and 10. The external Rayleigh number Ra_e is an indicator of the thermal irradiation strength. In general, the slope of the temperature gradient is directly proportional to the intensity of the external Rayleigh number. If $R^* = 0$, $f(Z)$ reduces to unity that corresponds to a liquid layer uniformly heated from below with no external thermal radiation incidence. For small optical thickness, Figure 2a, the incoming energy is absorbed throughout the layer. The temperature gradient corresponds to the case of uniform heat generation and deviates from unity with a smooth slope. However, the influence of the optical thickness on the temperature gradients has a more profound effect as it approaches to the optically thick limit (see Figure 2b). As $\tau \gg 1$, most of the radiative energy is absorbed in the upper strata. As a result, the temperature gradient is steep near the upper boundary and flat near the bottom of the liquid layer. The upper stratum is more unstable than the lower region, which has a more uniform temperature. The critical Marangoni number should, therefore, first decrease with τ for small optical thickness and then increase with τ. Generally, the critical Marangoni number is almost parabolic in nature when plotted as a function of the optical thickness. This trend is observed in the results shown below.

The Biot number Bi is a measurement of the effect of heat transfer due to conduction from the liquid layer at the interface to the convection in ambient gas. Figure 3 shows the influence of the Biot number on the critical Marangoni number. The convective heat transfer coefficient h is directly proportional to the Biot number. As h increases, the temperature variations at the upper surface become more smooth, and the change in surface tension becomes smaller, thus dampening the fluid motion. The critical Marangoni number, therefore, increases with the Biot number. The least stable situation exists when $\text{Bi} = 0$. This case corresponds to an insulated upper surface, and all incoming energy is retained and absorbed within the fluid layer. A destabilizing temperature gradient occurs throughout the layer, and the system becomes unstable. This aspect is further illustrated below.

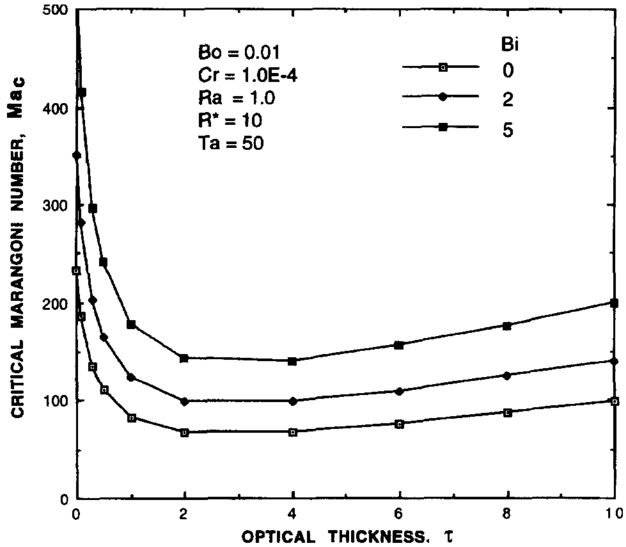
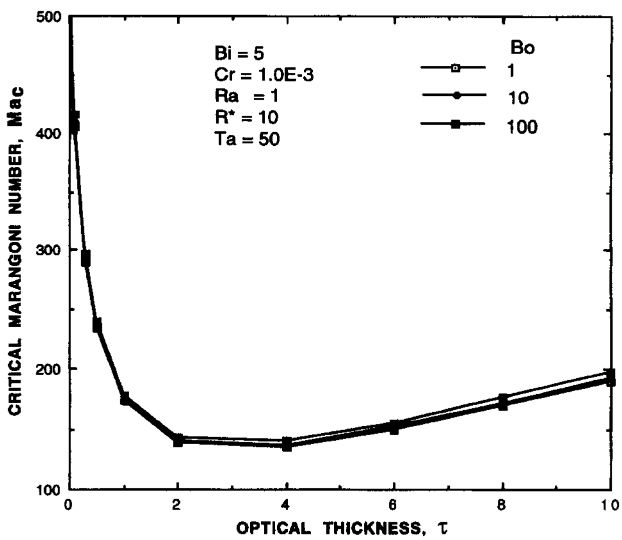


Figure 3 The effect of the thermal boundary condition on the critical Marangoni number

Most of the earlier work (Nield 1964; Vidal and Acrivos 1966; Namikawa et al. 1970; Friedrich and Rudraiah 1984) on the similar configuration is limited to a flat interface ($\text{Cr} = 0$), and thus the effect of the gravity wave can be eliminated in the analysis. However, as recently reported by Sarma (1985b, 1987), it is inappropriate to ignore the gravity effect in the linear model, because it predicts that the system is always unstable with respect to disturbances of small wave numbers and that there exists no critical Marangoni number. The Crispation and Bond numbers represent the measurement of the effects of interfacial curvature and interfacial gravity waves, respectively. The Crispation number represents the degree of deformability of the free surface and is inversely proportional to the mean surface tension. The Bond number is the ratio of the buoyancy and the capillary forces. In the present study, we consider the coupled effects of the Crispation and Bond numbers in order to illustrate the significance of the interfacial curvature ($\text{Cr} > 0$) and of gravity waves ($\text{Bo} > 0$) on convection. For a flat free surface without deflections ($\text{Cr} = 0$), only one single critical Marangoni number was observed for the present boundary conditions. However, two critical Marangoni numbers (with one zero and one nonzero critical wave number) were found along the neutral stability curve for wavy interface. These results agree with those of Smith (1966) and Sarma (1985b, 1987) when the presence of gravity waves is included in the model. A zero critical wave number (a_c) suggests that convective instability sets in above a finite threshold value Ma_c with infinite wavelength λ ($a = 2\pi/\lambda$).

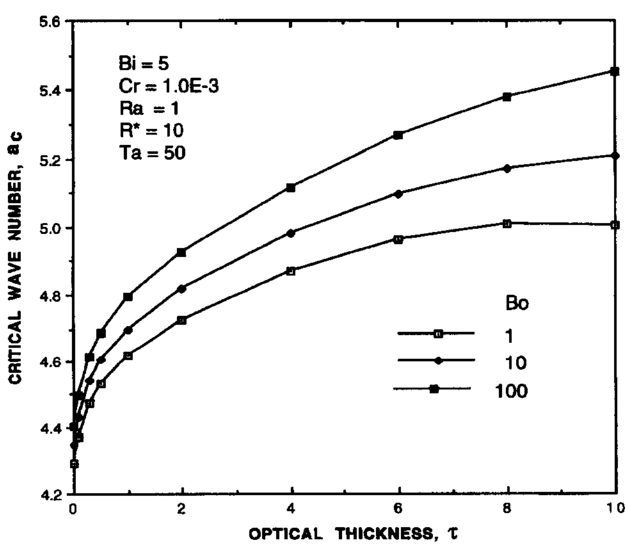
From a practical standpoint, it is always of interest to determine the onset of instability in finite cellular forms. Therefore, the following discussion is limited to the conditions leading to the onset of convective motion with finite wave-length character. To examine the effects of nonzero interfacial curvature two values for the Bond number (0.01 and 0.05) and the Crispation number (10^{-4} and 10^{-3}) were selected. However, the results show that the stability characteristics are not sensitive to Bo as long as a finite critical wave number ($a_c \gg 1$) is selected. This can be explained by referring to the boundary condition given by Equation 12. The influence of the Bond number on instability is minimal, because Bo is small compared to a_c. For larger Bo similar to those used by Sreenivasan and Lin (1978), Ma_c and a_c are shown in Figure 4a and b. Although the scale is compressed, the results indicate a stabilizing effect of the interfacial gravity waves.

The Crispation number is inversely proportional to the surface tension σ and becomes zero as $\sigma \to \infty$. As the surface tension decreases, less energy is required to generate convection. The critical Marangoni number decreases with increasing Crispation number. The influence of the Crispation number on the critical Marangoni number and the corresponding wave number



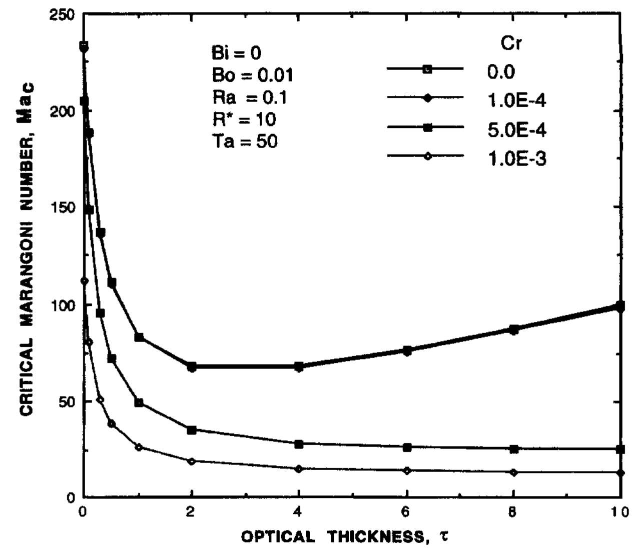
(a)

Figure 4a The effect of the buoyancy force and capillary force on the critical Marangoni number



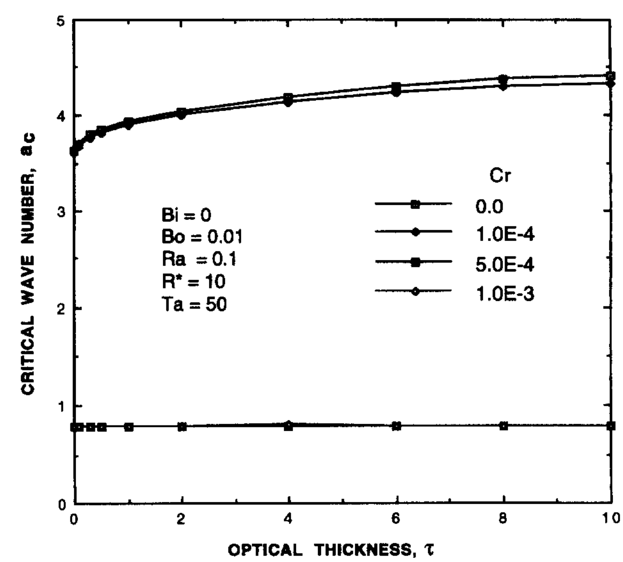
(b)

Figure 4b The effect of the buoyancy force and capillary force on the critical wave number



(a)

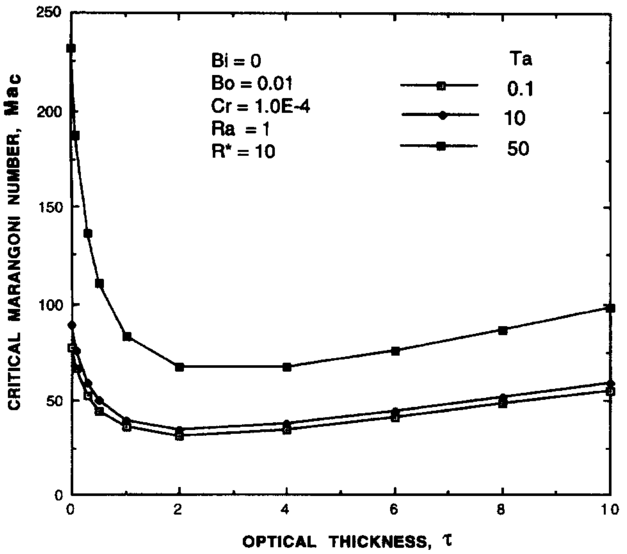
Figure 5a The effect of the upper surface deformability on the critical Marangoni number



(b)

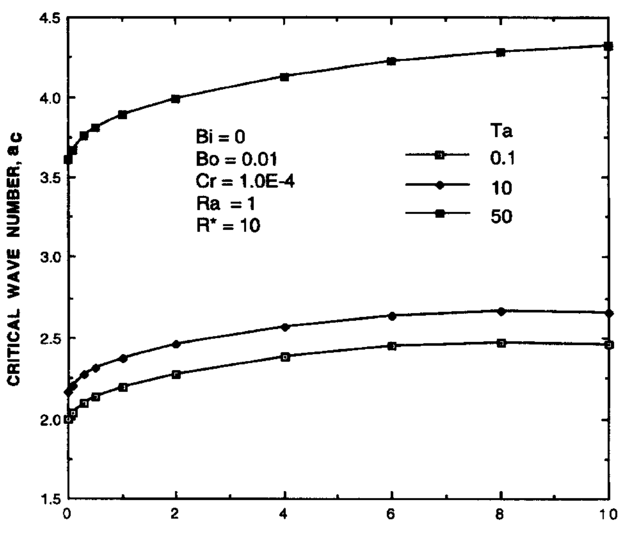
Figure 5b The effect of the upper surface deformability on the critical Marangoni number

is shown in Figures 5a and b. The results for $Cr = 0$ and 10^{-4} are essentially unchanged. However, for $Cr = 5 \times 10^{-4}$ and 10^{-3}, the entire behavior is profoundly influenced by interfacial curvature waves. It is also noticed that the critical wave number is also affected by the surface deflection. As the Crispation number increases, the convective motion has a tendency to set in with infinite wavelength. When the Crispation number is larger than 10^{-3}, the existence of a nonzero critical Marangoni number is no longer assured. It can be concluded that the approximation $Cr = 0$ is realistic for wave numbers that are not too small ($a \gg 1$) and Crispation numbers less than 10^{-2}. Outside these limits, the assumption of zero gravity waves in a linear model is not acceptable, because it predicts that the system is always unstable with small wave numbers disturbances and no critical



(a)

Figure 6a The effect of rotation on the critical Marangoni number



(b)

Figure 6b The effect of rotation on the critical wave number

Marangoni number. These findings are in accord with the conclusions of Cloot and Lebon (1985).

The Taylor number Ta represents the ratio of the Coriolis force to the viscous frictional force. The stabilizing effect of rotation on the critical Marangoni number and the corresponding wave number is shown in Figures 6a and b. The Coriolis force is unconditionally stabilizing, and Ma_c increases monotonically with the Taylor number.

The effects of external incident thermal irradiation on the critical Marangoni number are shown in Figure 7. The external and internal Rayleigh numbers Ra_e and Ra_i are the nondimensional measurement of the magnitudes of the radiative intensity incidence at the upper surface and the temperature difference across the liquid layer, respectively. As shown in Figure 7, the critical Marangoni number decreases with increasing the incident

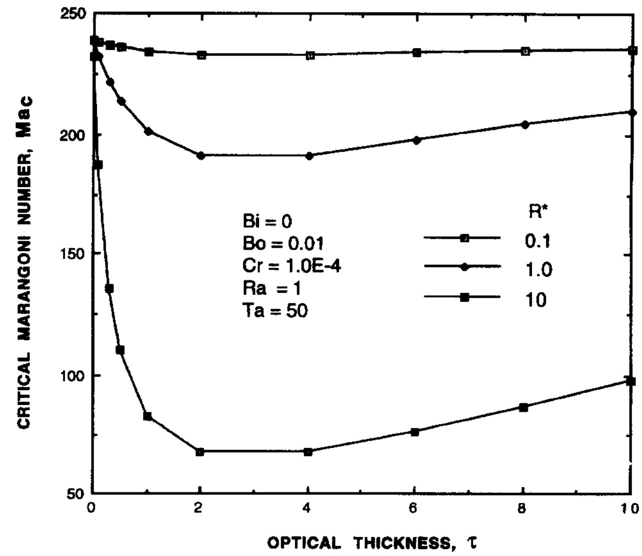


Figure 7 The effect of the external incident thermal radiation on the critical Marangoni number

thermal radiation intensity. Because most of the radiative energy is absorbed in the upper strata as $\tau \gg 1$, the temperature gradient is steep near the upper boundary and flat near the bottom of the liquid layer. The upper stratum is more unstable than the lower region, which has a more uniform temperature. Therefore, the critical Marangoni number is first decreased with τ for small optical thickness and then increased with large τ.

Finally, calculations are made to determine the effect of the internal Rayleigh number Ra_i on the critical Marangoni number. Ra_i is also a measure of the buoyancy force and the viscous force, and the Marangoni number represents the ratio of the surface tension force to viscous force. The ratio of these two dimensionless numbers provides an estimate of the relative magnitude of the surface tension and the buoyancy forces. The surface tension force becomes dominant compared to the buoyancy force in a microgravity environment. For low internal Rayleigh number, the effect on the Marangoni number is negligible. However, it is evident from Figure 8 that large internal

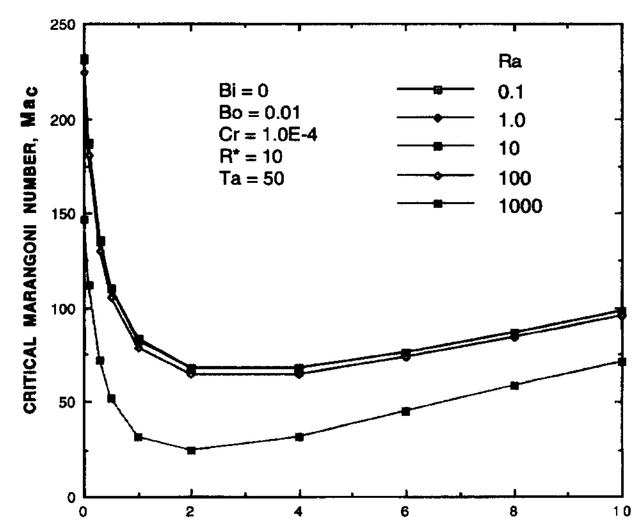


Figure 8 The effect of the internal buoyancy force on the critical Marangoni number

Rayleigh number plays a very significant role in ground-based convective instability experiment.

Final remarks

The conditions leading to the onset of Benard–Marangoni convective instability in a horizontal rotating fluid layer subjected to external thermal irradiation at the upper two-fluid interface has been determined by using the Chebyshev pseudospectral method. The dependence of the stability characteristics on the optical thickness, the Rayleigh, Taylor, Bond, Crispation, and Biot numbers is investigated. However, neither the long-wave nor the oscillatory mode have been considered in this study. They may play important roles in the present configuration. Further study is necessary to clarify their roles in convection.

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